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$$\frac{\partial}{\partial\beta} G_2 = -\frac{1}{\beta} G_2 + \frac{s}{2}\frac{\sin\sqrt{\beta}s}{\beta^{3/2}} = -\frac{1}{\beta} G_2 + \frac{s}{2\beta} G_1$$

$$= \frac{1}{2\beta}(s G_1 - 2 G_2)$$

$$\frac{\partial}{\partial\beta} G_3 = -\frac{1}{\beta} G_3 - \frac{1}{\beta}\frac{\partial G_1}{\partial\beta} = -\frac{1}{\beta} G_3 - \frac{1}{2\beta^2}(s G_0 - G_1)$$

So,

$$\boxed{\delta s = \frac{\delta\beta}{2\beta r}\left[2h - r_0 G_1 - r_0 s G_0 - r_0 \dot{r}_0 s G_1 - \frac{k G_1}{\beta} + \frac{k s G_0}{\beta}\right] - \frac{G_3 \delta k}{r} - \frac{G_1 \delta r_0}{r} - \frac{G_2 r_0}{r}\delta\dot{r}_0 - \frac{G_2 \dot{r}_0}{r}\delta r_0}$$

$$\delta\beta = \frac{2}{r_0}\delta k - \frac{2k}{r_0^2}\delta r_0 - 2\underline{v}_0\cdot\delta\underline{v}_0$$

$$\boxed{\delta\beta = \frac{2\delta k}{r_0} - \frac{2k}{r_0^3}\underline{r}_0\cdot\delta\underline{r}_0 - 2\underline{v}_0\cdot\delta\underline{v}_0}$$

$$f = 1 - \frac{k}{r_0} G_2$$

$$\delta f = -\frac{\delta k}{r_0} G_2 + \frac{k}{r_0^2}\delta r_0 G_2 - \frac{k}{r_0}\left(\underbrace{\frac{\partial G_2}{\partial s}}_{= G_1}\delta s + \underbrace{\frac{\partial G_2}{\partial\beta}}_{= \frac{1}{2\beta}(s G_1 - 2G_2)}\delta\beta\right)$$

$$\delta f = -\frac{\delta k}{r_0} G_2 + \frac{k}{r_0^2}\delta r_0 G_2 - \frac{k}{r_0}\left(G_1\delta s + \frac{1}{2\beta}(s G_1 - 2 G_2)\delta\beta\right)$$

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$$H = \sum_i \left[\frac{1}{2} m_i v_i^2\right] + \sum_i \sum_{j>i} \left[-\frac{G m_i m_j}{r_{ij}}\right]$$

$$\frac{m_i}{m_i+m_j} + \frac{m_j}{m_i+m_j} = 1$$

$$(\vec{v}_i - \vec{v}_j)^2 + (\vec{v}_i + \vec{v}_j)^2 = 2v_i^2 + 2v_j^2$$

$$\frac{1}{2}(m_i+m_j) V_{ij}^2 = \frac{1}{2}(m_i+m_j)\left(\frac{m_i \vec{v}_i + m_j \vec{v}_j}{m_i+m_j}\right)^2 = \frac{1}{2}\cancel{(m_i+m_j)} \frac{m_i^2 v_i^2 + m_j^2 v_j^2 + 2m_i m_j \vec{v}_i \cdot \vec{v}_j}{(m_i+m_j)^{\cancel{2}}}$$

$$\frac{1}{2} m_i v_i^2 + \frac{1}{2} m_j v_j^2 = \frac{1}{2} m_i (\vec{v}_i - \vec{v}_j)^2 + \frac{1}{2}(v_i + v_j)^2$$

$$H = \sum_i \frac{1}{2} m_i v_i^2 + \frac{1}{2} \sum_{j \neq i} \mu_{ij} \left[-\frac{G(m_i+m_j)}{r_{ij}}\right]$$

$$\frac{1}{2}\mu_{ij} v_{ij}^2 + \frac{1}{2}(m_i+m_j) \overset{\text{com}}{V_{ij}^2}$$

$$= \frac{1}{2} \frac{m_i m_j}{m_i+m_j}(v_i^2 + v_j^2 - \cancel{2m_i m_j \vec{v}_i \cdot \vec{v}_j}) + \frac{1}{2} \frac{m_i^2 v_i^2 + m_j^2 v_j^2 + \cancel{2m_i m_j \vec{v}_i \cdot \vec{v}_j}}{m_i+m_j}$$

$$= \frac{1}{2} m_i v_i^2 + \frac{1}{2} m_j v_j^2$$

?

$$\boxed{H = \sum_i \frac{1}{2} m_i v_i^2 + \frac{1}{2} \sum_{j \neq i} \left[\mu_{ij}\left(\frac{1}{2} v_{ij}^2 - \frac{G(m_i+m_j)}{r_{ij}}\right) + \frac{1}{2}(m_i+m_j) V_{ij}^2 - \frac{1}{2} m_i v_i^2 - \frac{1}{2} m_j v_j^2\right]}$$

This doesn't seem to agree w/ Goncalves-Ferrari et al... No, it does since this $= \frac{1}{2}\mu_{ij} \frac{1}{2} v_{ij}^2$!

But, it does agree w/ D.H.'s code!

Weird that it gives $-(N-2) \sum_i \frac{1}{2} m_i v_i^2$ KE terms.