The Outer Solar System

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0.1 Data

The chosen units are: masses relative to the sun, so that the sun has mass 1. We have taken $m_0 = 1.00000597682$ to take account of the inner planets. Distances are in astronomical units , times in earth days, and the gravitational constant is thus $G = 2.95912208286 \cdot 10^{-4}$.

planet	mass	
Jupiter	$m_1 = 0.000954786104043$	-3.5023653 -3.8169847 -1.5
Saturn	$m_2 = 0.000285583733151$	9.0755314 -3.0458353 -1.6
Uranus	$m_3 = 0.0000437273164546$	8.3101420 -16.2901086 -7.2
Neptune	$m_4 = 0.0000517759138449$	11.4707666 -25.7294829 -10.8
Pluto	$ m_5 = 1/(1.3 \cdot 10^8)$	-15.5387357 -25.2225594 -3.1

The data is taken from the book "Geometric Numerical Integration" by E. Hairer, C. Lubich and G. Wanner.

```
using Plots, OrdinaryDiffEq, DiffEqPhysics, RecursiveArrayTools
gr()

G = 2.95912208286e-4
M = [1.00000597682, 0.000954786104043, 0.000285583733151, 0.0000437273164546,
    0.0000517759138449, 1/1.3e8]
planets = ["Sun", "Jupiter", "Saturn", "Uranus", "Neptune", "Pluto"]

pos_x = [0.0,-3.5023653,9.0755314,8.3101420,11.4707666,-15.5387357]
pos_y = [0.0,-3.8169847,-3.0458353,-16.2901086,-25.7294829,-25.2225594]
pos_z = [0.0,-1.5507963,-1.6483708,-7.2521278,-10.8169456,-3.1902382]
pos = ArrayPartition(pos_x,pos_y,pos_z)

vel_x = [0.0,0.00565429,0.00168318,0.00354178,0.00288930,0.00276725]
vel_y = [0.0,-0.00412490,0.00483525,0.00137102,0.00114527,-0.00170702]
vel_z = [0.0,-0.00190589,0.00192462,0.00055029,0.00039677,-0.00136504]
vel = ArrayPartition(vel_x,vel_y,vel_z)

tspan = (0.,200_000)
```

```
(0.0, 200000)
```

The N-body problem's Hamiltonian is

$$H(p,q) = \frac{1}{2}\sum_{i=0}^{N}\frac{p_i^T p_i}{m_i} - G\sum_{i=1}^{N}\sum_{j=0}^{i-1}\frac{m_i m_j}{\|q_i - q_j\|}$$

Here, we want to solve for the motion of the five outer planets relative to the sun, namely, Jupiter, Saturn, Uranus, Neptune and Pluto.

```
const ∑ = sum
const N = 6
potential(p, t, x, y, z, M) = -G*∑(i->∑(j->(M[i]*M[j])/sqrt((x[i]-x[j])^2 +
    (y[i]-y[j])^2 + (z[i]-z[j])^2), 1:i-1), 2:N)
```

```
potential (generic function with 1 method)
```

0.2 Hamiltonian System

`NBodyProblem` constructs a second order ODE problem under the hood. We know that a Hamiltonian system has the form of

$$\dot{p} = -H_q(p,q) \quad \dot{q} = H_p(p,q)$$

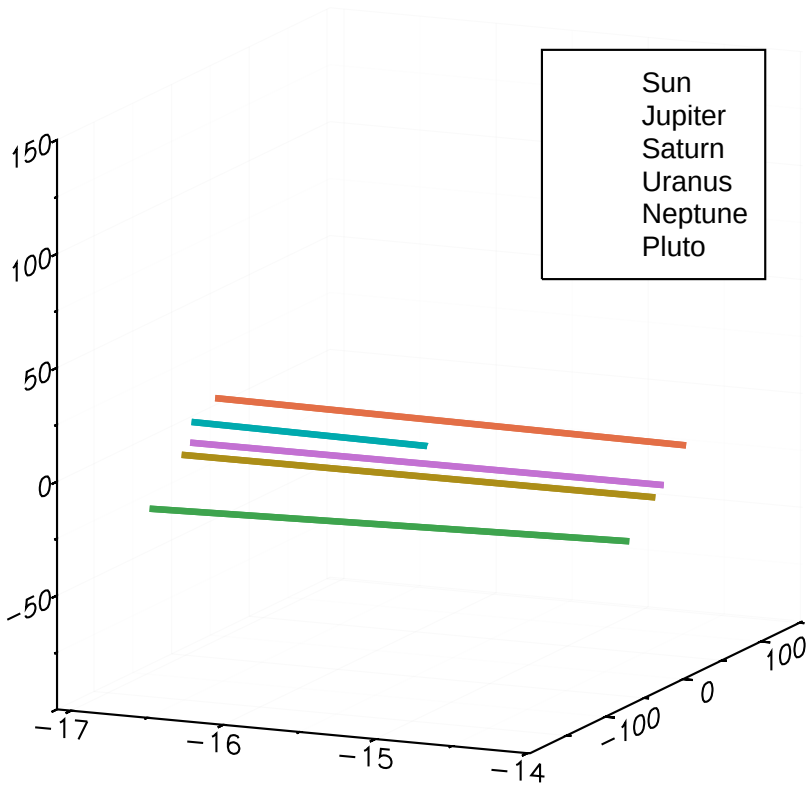
For an N-body system, we can symplify this as:

$$\dot{p} = -\nabla V(q) \quad \dot{q} = M^{-1}p.$$

Thus $\dot{q}$ is defined by the masses. We only need to define $\dot{p}$, and this is done internally by taking the gradient of V. Therefore, we only need to pass the potential function and the rest is taken care of.

```
nprob = NBodyProblem(potential, M, pos, vel, tspan)
sol = solve(nprob,Yoshida6(), dt=100);
```

```
orbitplot(sol,body_names=planets)
```



0.3 Appendix

This tutorial is part of the DiffEqTutorials.jl repository, found at: https://github.com/JuliaDiffEq/DiffEq

To locally run this tutorial, do the following commands:

```
using DiffEqTutorials
DiffEqTutorials.weave_file("models","07-outer_solar_system.jmd")
```

Computer Information:

```
Julia Version 1.1.1
Commit 55e36cc308 (2019-05-16 04:10 UTC)
Platform Info:
  OS: Linux (x86_64-pc-linux-gnu)
  CPU: Intel(R) Core(TM) i7-3770 CPU @ 3.40GHz
  WORD_SIZE: 64
  LIBM: libopenlibm
  LLVM: libLLVM-6.0.1 (ORCJIT, ivybridge)
```

Package Information:

```
Status `~/.julia/environments/v1.1/Project.toml`
[7e558dbc-694d-5a72-987c-6f4ebed21442] ArbNumerics 0.5.4
```

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```