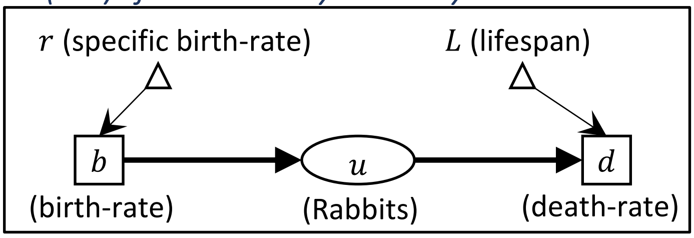
200. Dynamical narratives: How do systems develop over time?

Fundamental Principle of *system dynamics* (*SD*): *Narrative emerges from structure!*

Structure-process diagram (SPD) of the Rabbits dynamical system:



Designing the Rabbits dynamical system

Stocks (or ***state variables***):

$u(t)$, where $u_0 \equiv u(0) = 1000$ rabbit (*Here, units are important!*)

Dynamical ***processes***:

$$\dot{u} \equiv \frac{du}{dt} = (b - d)\ \text{rabbit/month}$$

Structural ***relations***:

$$b = r\, u$$
$$d = u/L$$

Numerical ***parameters***:

$$r \equiv \frac{\dot{u}}{u} = \underline{\hspace{3em}}\ (\text{month})^{-1}$$
$$L = 48\ \text{month}$$

Differential equations (*DE*):

$$\left.\begin{aligned} \dot{u} &= (r - L^{-1})\, u \\ u_0 &= 1000 \\ r &= \underline{\hspace{4em}} \\ L &= 48 \end{aligned}\right\}\ \text{(From now on, we will ignore units for convenience!)}$$

Implementing the Rabbits narrative

Behaviour over time (*BOT*):

$$u(t) = \int_{\tau=0}^{\tau=t} \dot{u}\, dt = \int_{\tau=0}^{\tau=t} (r - L^{-1})\, u\, dt$$

In general, biological systems are *never* exactly integrable, but we have simplified the Rabbits system so greatly (by ignoring migration and predation) that we can calculate this integral exactly:

$$\frac{du}{dt} = (r - L^{-1})\, u$$
$$\Rightarrow \qquad \int \frac{du}{u} = \int (r - L^{-1})\, dt$$
$$\Rightarrow \qquad \ln u = (r - L^{-1})\, t + c$$
$$\Rightarrow \qquad u = \exp(c) \cdot \exp\big((r - L^{-1})\, t\big)$$
$$\Rightarrow \qquad u(t) = u_0 \exp\big((r - L^{-1})\, t\big) = 1000\, e^{\underline{\hspace{2em}}\, t}$$

We are *very* lucky: This is probably the *only* time we will be able to integrate our DEs exactly! From now on, we will need to calculate BOTs *numerically* using the julia DynamicalSystems package.